COMPARISON BETWEEN RECTANGULAR AND ANNULAR WAVEGUIDE MODAL ANALYSES FOR WAVEGUIDE BENDS*

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Abstract

Coupling in waveguide bends of nonuniform curvature is determined by employing modal expansions for rectangular and annular waveguides. For large curvature waveguide bends an iterative method is suitable only for the annular modal analysis.

Introduction

Using a method attributed to Schelkunoff[1], the electromagnetic fields in nonuniform waveguides are derived by converting Maxwell's equations into a complete set of coupled transmission-line equations referred to as "generalized telegraphist's equations." The dependent variables in these first-order coupled differential equations are the forward and backward waveguide mode amplitudes a_n and b_n respectively.

The method is based on the expansion of the transverse components of the electromagnetic fields at any cross-section of the nonuniform waveguide in terms of a complete modal expansion. Thus, for waveguide bends of rectangular cross-section, it is possible to express the fields at any cross-section normal to the center line of the waveguide, in terms of local rectangular waveguide modes,[2] or in terms of local annular waveguide modes.[3]

The solutions of the electromagnetic fields at any cross-section should not depend on the particular modal analysis used, however the set of coupled differential equations for the wave amplitudes depend upon the modal expansion. In general, the coupling coefficients in these equations depend upon the center line coordinate of the waveguide. Thus unless the power associated with the spurious modes at any cross-section is small compared to the incident power it is necessary to employ sophisticated numerical methods to derive satisfactory solutions to the coupled differential equation. However if the power associated with the spurious modes at any cross-section of the waveguide is small, a simple first-order iterative approach to solve the coupled differential equations is found to be very suitable.

Illustrative Examples

H-plane waveguide bends with sinusoidal shaped center lines and uniform rectangular waveguide ports are analyzed using both the rectangular and the annular modal expansions (See insert Fig. 1). The incident field at the input port ($\xi=0$) is assumed to be the principal $TE_{1,o}$ mode of unit amplitude, $a_1(o)=1$. The basis functions are normalized such that the incident power in this case is unity.[4] The width of the waveguide is $a=1.75\lambda$ and the centerline of the waveguide bend is given by the expression

$$h(\xi) = (w/\pi)\sin(\pi\xi/w), \qquad (1)$$

thus we are considering 90° bends. A fourth-order Runge-Kutta numerical method is used to solve the resulting coupled differential equations and these are compared with the first-order iterative solutions.

In Figs. 1 and 2 first-order iterative solutions for the complex amplitude of the $TE_{2,o}$ mode, $a_2(\xi)$, are plotted as a function of the normalized distance ξ/w.

$$a_2(\xi)=a_1(o)\exp\{-i\int_o^\xi K_2(u)du\}\int_o^\xi S_{21}(u)\cdot\exp\{i\int_o^u [K_2(w)-K_1(w)]dw\}du \qquad (2)$$

where K_n is related to the propagation coefficients and $S_{21}(u)$ is the $TE_{1,o}$ to $TE_{2,o}$ mode coupling coefficient for forward propagating waves. For waveguide bends with small curvatures (w/a=14, Fig. 1), both the iterative solutions (using the rectangular and the annular modal analysis) agree with the Runge-Kutta solution. However for waveguide bends with large curvatures (w/a=6, Fig. 2) the power associated with the spurious modes near the center of the bend is larger than the power retained in the incident mode if the rectangular modal expansion is used. Thus the iterative approach fails in these cases. But, for the same cases, the iterative method is suitable provided that the annular modal expansion is used. Since local elementary annular waveguides fit the geometry of the waveguide bends better than elementary rectangular waveguides it is shown that mode coupling is significantly smaller when the annular modal expansion is used. Hence using the annular modal analysis the power associated with the spurious modes is small compared to the incident power at any transverse plane of the waveguide bend.

In Figs. 3 and 4 we plot the loci of the phasor $A_2(v)$, where

$$A_2(v)=a_1(o)\exp\{-i\int_o^w K_2(u)du\}\int_o^v S_{21}(u)\cdot\exp\{i\int_o^u [K_2(w)-K_1(w)]dw\}du \qquad (3)$$

and $\Delta A_2=A_2(v+\Delta)-A_2(v)$ represents the contribution to the spurious mode amplitude $a_2(w)$ originating from the section of waveguide between $\xi=v$ and $\xi=v+\Delta v$ (measured along the center line). At the origin (Fig. 3 and 4) v=0 and at the end point of the loci v=w, hence $A_2(w)=a_2(w)$. Figures 3 and 4 graphically show that the coupling per unit length of the waveguide bend is much larger when the rectangular modal analysis is used as compared with the annular modal analysis. However, as a result of destructive interference, the net contribution to mode $TE_{2,o}$ at the output of the waveguide, as derived from both analyses, is the same.

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References

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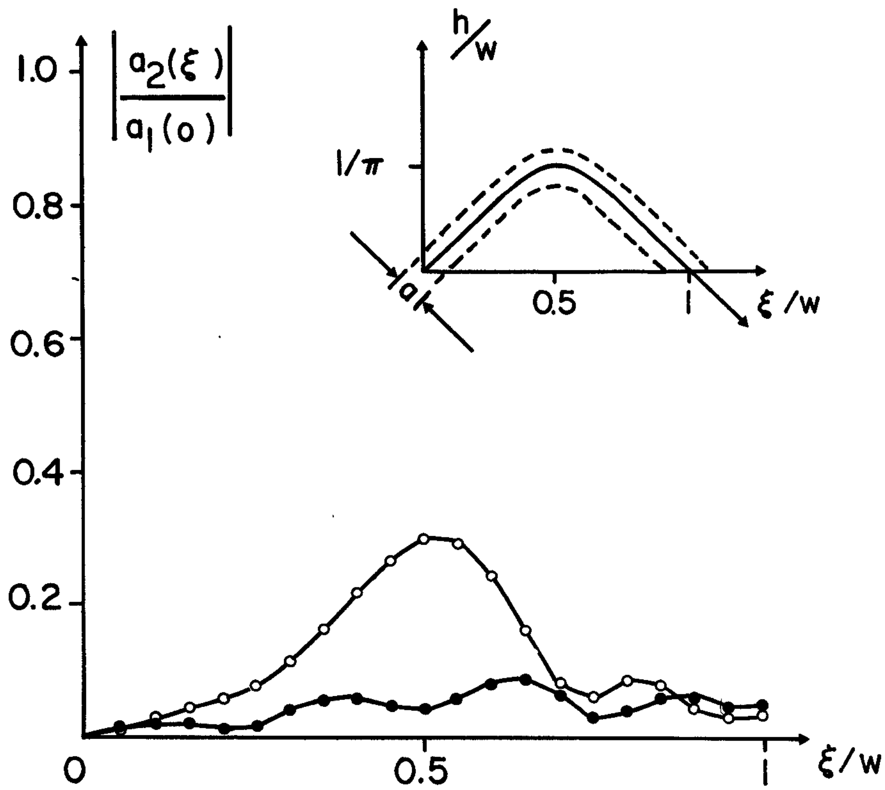

Fig. 1 First-order iterative solutions for the $TE_{2,o}$ mode amplitude, $a_2(\xi)$. Circles, rectangular modal analysis; crosses, annular modal analysis, w/a=14.

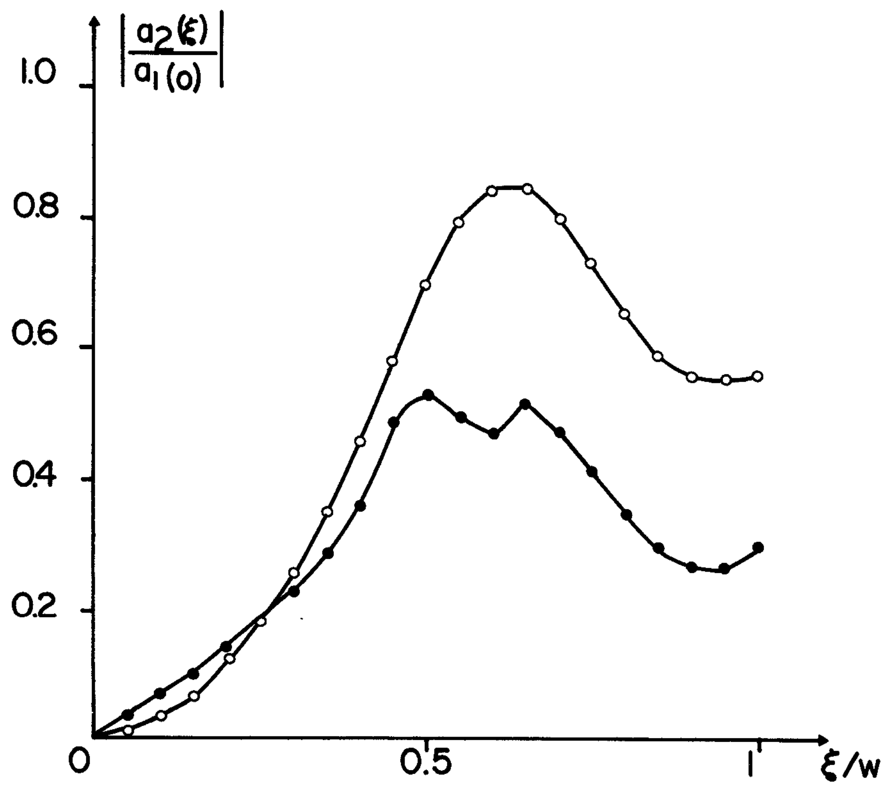

Fig. 2 First-order iterative solutions for the $TE_{2,o}$ mode amplitude, $a_2(\xi)$. Circles, rectangular modal analysis; crosses, annular modal analysis, w/a=6.

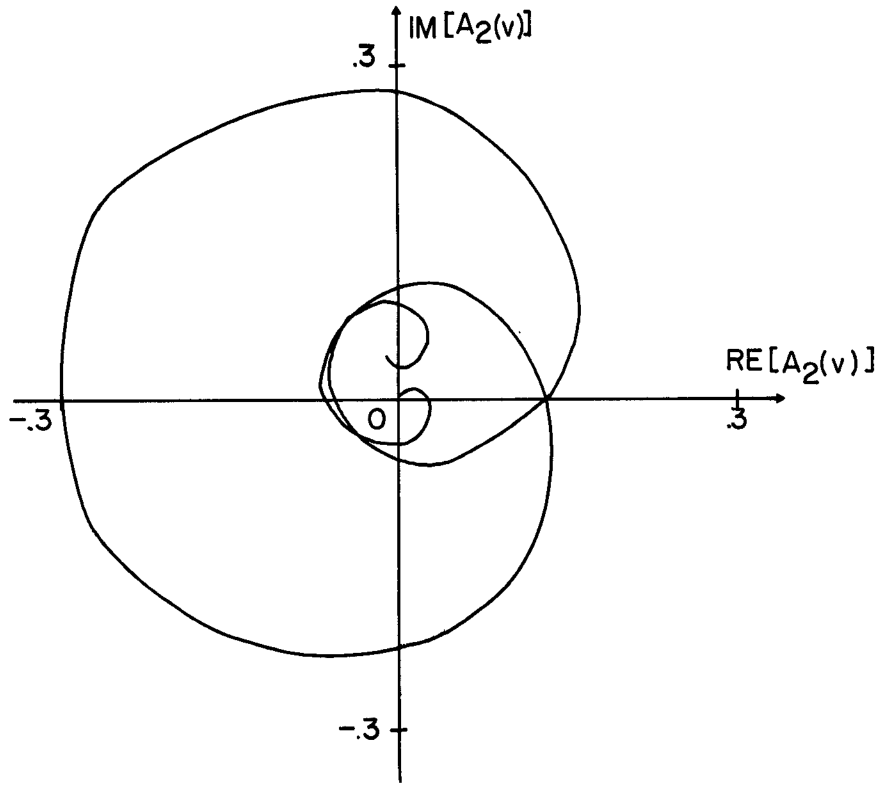

Fig. 3 Loci of $A_2(v)$ using the rectangular modal analysis.

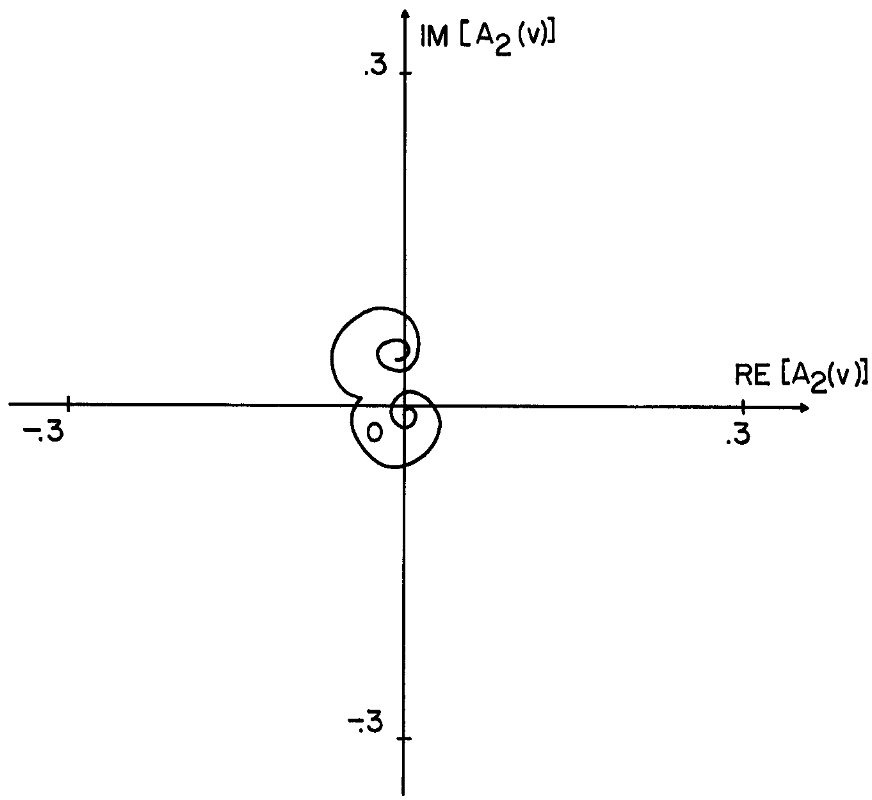

Fig. 4 Loci of $A_2(v)$ using the annular modal analysis.